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NPTEL ONLINE
CERTIFICATION COURSE

STABILITY AND STEADY STATE ERROR

DEFINITION OF STABILITY

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STABILITY

Three requirements enter into the design of a control system:

- Transient response (TR)
- Stability (S)
- Steady-state errors (SSE)



STABILITY –Natural Response

- A linear time-invariant (LTI) system is **STABLE** if the natural response approaches zero as time approaches infinity
- A linear time-invariant (LTI) system is **UNSTABLE** if the natural response grows without bound as time approaches infinity
- A LTI system is **MARGINALLY STABLE** if the natural response neither decays nor grows but remains constant or oscillates as time approaches infinity



STABILITY – Total Response

- A system is STABLE if every bounded input yields a bounded output
- A system is UNSTABLE if any bounded input yields an unbounded output
- From the perspective of the time response plot of a physical system, instability is displayed by transients that grows without bound and, consequently a total response that does not approach a steady state value

STABILITY – Location of Poles

- STABLE systems have closed-loop transfer functions with poles only in the left-half plane
- UNSTABLE systems have closed-loop transfer functions with at least one pole in the right-half plane and/or poles of multiplicity greater than one on the imaginary axis
- marginally STABLE systems have closed-loop transfer functions with only imaginary axis poles of multiplicity one and poles in the left-half plane

ROUTH-HURWITZ CRITERION FOR STABILITY

- A method that yields stability information without the need to solve for the poles.
- Using this method, we can tell how many closed-loop system poles are in the left-half plane, in the right-half plane, and on the $j\omega$ -axis
- Generate and interpret the Routh table



ROUTH TABLE

- Begin by labeling the rows with powers of s from the highest power of the denominator to s^0
- Start with the coefficient of the highest power of s in the denominator and list, horizontally in the first row, every other coefficient
- In the second row, list horizontally, starting with the next highest power of s , every coefficient that was skipped in the first row
- The remaining entries are filled in as follows:
- Each entry is a negative determinant of entries in the previous two rows divided by the entry in the first column directly above the calculated row
- The left-hand column is always the first column of the previous two rows
- The right-hand column is the elements of the column above and to the right
- The table is complete when all of the rows are completed down to s^0 .

ROUTH TABLE

- The number of roots of the polynomial that are in the right half plane is equal to the number of sign changes in the first column
- A system is STABLE if there are no sign changes in the first column of the Routh table



REFERENCE BOOKS

- Norman S. Nise, Control Systems Engineering, Wiley, 2013
- Katsuhiko Ogata, Modern Control Engineering, Prentice Hall, 2010.

